

CIPFA FORMULAE

For Certificate, Diploma & FTPC Examinations

Mean

Population (ungrouped)

$$\mu = \frac{\sum x}{N}$$

Population (grouped)

$$\mu = \frac{\sum fx}{\sum f}$$

Sample (ungrouped)

$$\bar{x} = \frac{\sum x}{n}$$

Sample (grouped)

$$\bar{x} = \frac{\sum fx}{\sum f}$$

Standard Deviation

Population (ungrouped)

$$\sigma = \sqrt{\frac{\sum (x - \mu)^2}{N}}$$

Population (grouped)

$$\sigma = \sqrt{\frac{\sum f(x - \mu)^2}{\sum f}}$$

Sample (ungrouped)

$$s = \sqrt{\frac{\sum (x - \bar{x})^2}{n - 1}}$$

Sample (grouped)

$$s = \sqrt{\frac{\sum fx^2}{\sum f - 1} - (\bar{x})^2} \quad \text{OR} \quad s = \sqrt{\frac{\sum f(x - \bar{x})^2}{\sum f - 1}}$$

Sampling and sampling distributions

Standard error of the mean

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} \quad \text{OR} \quad s_{\bar{x}} = \frac{s}{\sqrt{n}}$$

Where s is used as an estimate of σ

Hypothesis tests

Large sample test for μ

$$z = \frac{\bar{x} - \mu}{\sigma_{\bar{x}}} \quad \text{OR} \quad z = \frac{\bar{x} - \mu}{s_{\bar{x}}}$$

Small sample test for μ

$$t = \frac{\bar{x} - \mu}{\sigma_{\bar{x}}} \quad \text{OR} \quad t = \frac{\bar{x} - \mu}{s_{\bar{x}}}$$

χ^2 Test Statistic

$$\chi^2 = \sum \frac{(O - E)^2}{E}$$

Regression and Correlation

Correlation coefficient

$$r = \frac{n\sum xy - \sum x \sum y}{\sqrt{(n\sum x^2 - (\sum x)^2)}\sqrt{(n\sum y^2 - (\sum y)^2)}}$$

OR

$$r = \frac{\sum xy - n\bar{x}\bar{y}}{\sqrt{(\sum x^2 - n\bar{x}^2)(\sum y^2 - n\bar{y}^2)}}$$

Coefficient of determination

$$r^2$$

Single regression (least squares)

$$b = \frac{n\sum xy - \sum x \sum y}{n\sum x^2 - (\sum x)^2}$$

$$a = \frac{\sum y}{n} - \frac{b\sum x}{n}$$

Time-related discounting

(a) Simple interest

$$S_n = p + nrp \quad \text{Where} \quad \begin{array}{l} p = \text{the original amount invested} \\ r = \text{interest rate} \\ n = \text{number of periods} \\ S_n = \text{the sum after } n \text{ periods} \end{array}$$

(b) Compound interest

$$S_n = p(1+r)^n \quad \text{Where} \quad \begin{array}{l} p = \text{principal} \\ r = \text{interest rate} \\ n = \text{number of periods} \\ S_n = \text{the sum invested after } n \text{ periods} \end{array}$$

(c) Annual percentage rate

$$\text{APR} = (1 + r/n)^n - 1$$

where r = annual rate
 n = number of periods in year

(d) Daily percentage rate

$$K = \sqrt[D]{1+R} - 1$$

Where

K is the daily interest rate

R is the rate given

D is the number of days it relates to

(e) Discount factor

$$PV = S_n \frac{1}{(1+r)^n} \quad \text{Where} \quad \begin{array}{l} PV = \text{present value} \\ r = \text{interest rate} \\ n = \text{number of periods} \\ S_n = \text{the sum invested after } n \text{ periods} \end{array}$$

(f) Internal rate of return by interpolation

$$IRR = A + \frac{C}{C-D} \times (B - A) \quad \text{Where}$$

A = lower discount rate
 B = higher discount rate
 C = NPV at lower discount rate
 D = NPV at higher discount rate

(g) Real rate of interest

$$\text{Real rate} = \frac{(1 + \text{nominal rate})}{(1 + \text{the inflation rate})} - 1$$

(h) Equity cost of capital

Dividend valuation model:

$$r = \frac{d}{MV} \quad \text{Where}$$

r = shareholder's cost of capital
 d = annual dividend per share
 MV = ex dividend share price

Dividend growth model:

$$r = \frac{d_0(1 + g)}{MV} + g \quad \text{Where}$$

r = the shareholders cost of capital
 d_0 = current dividend (net)
 g = expected annual growth in dividend payments (%)
 MV = current ex div market price

(i) Cost of debt capital and preference shares

$$i = \frac{K}{W_o} \quad \text{Where}$$

W_o = current market price of debt capital
 K = the interest (dividend) received
 i = the cost of debt (preference share) capital

(j) Present value of future dividends

$$P_0 = \frac{D_1}{i - g} \quad \text{Where} \quad P_0 = \text{present share price}$$

$D_1 = \text{dividend per share received at end of year 1}$
 $i = \text{discount rate}$
 $g = \text{expected annual percentage rate of growth}$

(k) Capital asset pricing model

Can be expressed as:

$$R_s = R_{rf} + \beta(R_{rm} - R_{rf}) \quad \text{or} \quad ER_j = R_f + (ER_m - R_f)B$$

Where R_s or ER_j = the expected return/cost of capital

R_{rf} or R_f = the risk free rate of return

R_{rm} or ER_m = the return on the market portfolio

β or B = the Beta factor of the individual security

(l) Standard deviation of NPV

$$V_p = \sum [V(X_i)]^2 P(X_i) \quad \text{where } V_p = \text{variance of the project's NPV}$$

$V(X_i) = \text{variance of the NPV of outcome } X_i \text{ from the expected NPV}$
 $P(X_i) = \text{probability of outcome } X_i$

Stock control

Economic Order Quantity

$$q = \sqrt{\frac{2 C D}{H}} \quad \text{Where} \quad q = \text{Economic Order Quantity}$$

C = order cost

D = annual demand

H = stock holding cost per unit per annum

Reorder level model

Reorder level (ROL) = Lead time x Demand per time unit + safety stock

Miller-Orr model

$$Z = \sqrt[3]{\frac{3 F \sigma^2}{4 K}} + L \quad \text{Where} \quad K = \text{daily opportunity cost of cash}$$

F = transaction cost

σ^2 = variance of net daily cash flows

Z = target balance

H = upper limit

L = Lower limit

$$H = 3Z - 2L$$
$$W = \frac{(4Z - L)}{3}$$

W = average balance

Baumol Model - Optimal cash injection

$$C = \sqrt{\frac{2 b t}{i}} \quad \text{Where} \quad C = \text{cash injection}$$

b = brokers' fees

t = net cash flow over the period

i = opportunity cost of interest forgone or cost of borrowing to provide funds